$\Rightarrow \frac{1}{2} \leq U_{n+1} \leq \frac{e}{3}$

$\Longrightarrow 0 \leq U_{n+1} \leq 1$

Conclusion : $0 \leq U_n \leq 1$, pour tout $n \in IN$

3) a) $\int_{\alpha}^{x} f'(v)dv = [f(v)]_{\alpha}^{x} = f(x) - f(\alpha)$

b) $x \in [0, 1]$

$$| f(x) - f(\alpha) | = | \int_{\alpha}^{x} f'(v)dv |$$

*Pour $x \geq \alpha$:

$$| f(x) - f(\alpha) | \leq \int_{\alpha}^{x} |f'(v)|dv$$

$$\Rightarrow | f(x) - f(\alpha) | \leq \int_{\alpha}^{x} \frac{e}{4} dv$$

$$\Rightarrow | f(x) - f(\alpha) | \leq \frac{e}{4}(x - \alpha)$$

*Pour $x < \alpha$:

$$| f(x) - f(\alpha) | = | \int_{x}^{\alpha} f'(v)dv |$$

$$\Rightarrow | f(x) - f(\alpha) | \leq \int_{x}^{\alpha} |f'(v)|dv$$

$$\leq \int_{x}^{\alpha} \frac{e}{4} dv \leq \frac{e}{4}(\alpha - x)$$

D'où

$$|f(x) - f(\alpha)| \leq \frac{e}{4} |x - \alpha| \quad \forall x \in [0, 1]$$

$U_n \in [0, 1] \Rightarrow | f(U_n) - f(\alpha) | \leq \frac{e}{4} |U_n - \alpha|$

$$\Rightarrow | U_{n+1} - \alpha | \leq \frac{e}{4} |U_n - \alpha|$$

c) *pour $n = 0$

$$| U_0 - \alpha | = | \frac{1}{2} - \alpha | \leq \frac{1}{2}$$

Car $\alpha \in [0, 1]$

D'où $| U_0 - \alpha | \leq \frac{1}{2} (\frac{e}{4})^0$

(Vrai)

* supposons que :

$$| U_n - \alpha | \leq \frac{1}{2} (\frac{e}{4})^n$$

et montrons que

$$| U_{n+1} - \alpha | \leq \frac{1}{2} (\frac{e}{4})^{n+1}$$

D'après b)

$$| U_{n+1} - \alpha | \leq \frac{e}{4} | U_n - \alpha |$$ et comme

$$| U_n - \alpha | \leq \frac{1}{2} (\frac{e}{4})^n$$

alors $| U_{n+1} - \alpha | \leq \frac{1}{2} (\frac{e}{4})^{n+1}$

Conclusion : $| U_n - \alpha | \leq \frac{1}{2} (\frac{e}{4})^n$; $n \in IN$

Or $\frac{e}{4} \in]-1, 1[\Rightarrow \lim_{n \to +\infty} \frac{1}{2} \left(\frac{e}{4}\right)^n = 0$ et par suite $\lim_{n \to +\infty} U_n = \alpha$

EX 25 :

1) $\lim_{x \to +\infty} (\sqrt{3})^x = +\infty$ car $\sqrt{3} > 1$

2) $\left(\frac{1}{2}\right)^{-x} = 2^{x-1} = \frac{1}{2} \times 2^x$

$\lim_{x \to +\infty} \left(\frac{1}{2}\right)^{-x} = \lim_{x \to +\infty} (\frac{1}{2} \times 2^x) = +\infty$ car $2 > 1$

3) $\frac{2^x}{2^{x^2+x}} = \frac{2^x}{2^x . 2^{x^2}} = \frac{1}{2^{x^2}} = 2^{-x^2}$

$\lim_{x \to +\infty} \frac{2^x}{2^{x^2+x}} = \lim_{x \to +\infty} 2^{-x^2}$

$X = -x^2$ $\lim_{x \to -\infty} 2^X = 0$

Qarray.tn - Agra Blech- Qarray.tn